period of observation there was hardly a day during which there were not one or more large spots on the sun.

Acknowledgment

The assistance of Mr. R. W. N. Mason in the construction and operation of the recording equipment is acknowledged. The work described above was carried out as part of the programme of the Radio Research Board. This paper is published by permission of the Director of the National Physical Laboratory and the Director of Radio Research of the Department of Scientific and Industrial Research.

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COMPOSITE LADDER FILTERS

Second-Order Image Impedances

By R. O. Rowlands, M.Sc., A.M.I.E.E.

(*B.B.C. Engineering Division*)

SUMMARY.—The first part of the paper indicates a method of designing composite ladder-type filters with equal or inverse impedance functions of the second order which are independent of the propagation function. Complete design formulae are given for filters comprising one, two, and three half-sections.

The filters described in the second part have an impedance function of the second order at one pair of terminals only.

The advantage of these systems of design is that the filters so obtained have less components than those designed by any other method and are therefore more economical to construct.

1. Introduction

THE image impedance of a filter is a function of frequency. An impedance function of the first order or constant-k varies considerably over the pass band, whereas an impedance of the second order or m-derived can be made relatively flat over a large part of the pass band by a suitable choice of m. The number of elements in a filter section increases with the order of its image impedance. In designing a composite filter with second-order image impedances it is customary, therefore, for the sake of economy to design the bulk of the filter with cascaded sections having an image impedance of the first order, and to terminate the filter with half-sections having an image impedance of the first order on one side and of the required second order on the other side. An example of this is shown in Fig. 1.

If the impedance function of the terminating half-sections is prescribed, this determines their attenuation function, as the two functions are interrelated, and unless this attenuation function is sufficiently alike to one of the sections in the bulk of the filter for the latter to be omitted, the process of terminating a filter in a second-order impedance function adds to the cost of the filter. This is particularly the case when there are only a few sections in the bulk of the filter because, first, there is less likelihood of employing the attenuation of the terminating half-sections to supersede a section in the bulk of the filter and, secondly, the smaller the bulk of the filter the larger is the termination in proportion to the whole.

Starting with a filter designed in this manner, however, it will be shown how the half sections, which are connected to each end of the filter to get the correct impedance characteristic, may be brought together at one end of the filter and then removed without altering the filter impedance.

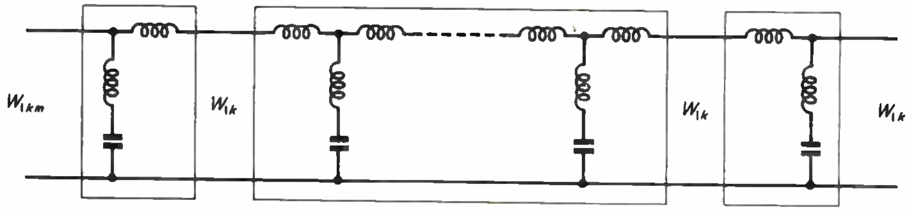


Fig. 1. *Composite filter of 1st-order sections terminated by 2nd-order half-sections.*

2. Notation

The following notation will be used:—

MS accepted by the Editor, March 1951

$m_1, m_2, ---$ = m values of sections used for their contribution to the attenuation

m_a = m value of half-sections used because of their impedance properties to terminate the filter

W'_{1k} = 1st order mid-series image impedance

W'_{2k} = 1st order mid-shunt image impedance

W'_{1km} = 2nd order mid-series image impedance

W'_{2km} = 2nd order mid-shunt image impedance

z = reactance of the series arm of a general *half*-section constant-k filter

y = susceptance of the shunt arm of a general *half*-section constant-k filter

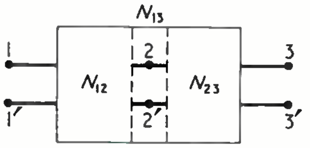
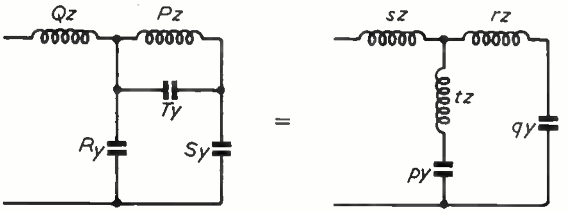


Fig. 2 (right). *Network N_{13} split into two N_{12} and N_{23}.*

Fig. 3 (below). *Two equivalent networks.*



3. Theorem and Transformation Formulae

Use will be made of a theorem and transformation formulae derived by the author in two previous papers.* The theorem states that if the 4-terminal network N_{13} of Fig. 2 can be split into two cascade-connected 4-terminal networks N_{12} and N_{23} in such a way that both N_{12} and N_{13} have equal image impedances at terminals 1,1′ then N_{12} and N_{23} have equal image impedances at terminals 2,2′.

The transformation formulae which have been slightly modified from the original are:

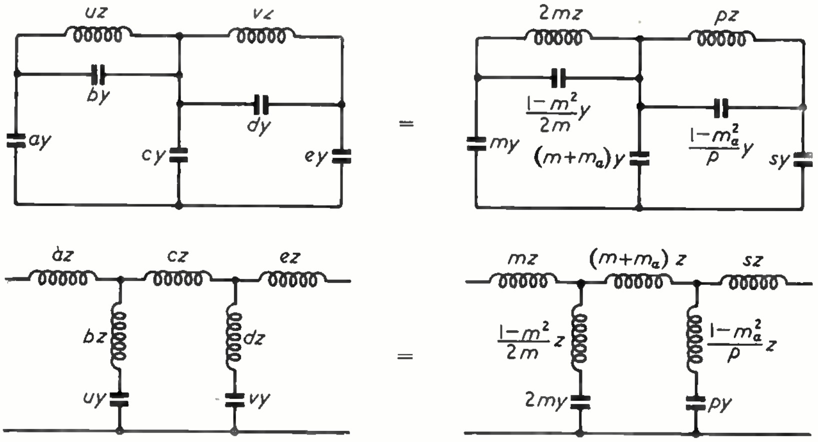


Fig. 4 (right). *Two pairs of equivalent networks.*

I.

The networks of Fig. 3 are equivalent if:

$$P = \frac{r^2}{r+t} \qquad Q = \frac{rt}{r+t} + s \qquad R = \frac{p\,(r+t)}{r}$$

$$S = \frac{qr - pt}{r} \qquad T = \frac{pt\,(r+t)}{r^2}$$

In particular if

$q = 1$, $r = 1 + m$, and $pt = 1 - m^2$, then:

$$P = \frac{(1+m)p}{p+1-m} \qquad Q = \frac{1-m^2}{p+1-m} + s$$

$$R = p + 1 - m \qquad S = m$$

$$T = \frac{(1-m)\,(p+1-m)}{p}$$

The same formulae hold when small letters and capitals are interchanged.

II.

The networks of Fig. 4 are equivalent when:

$$a = m_a \qquad b = \frac{(1 - m_a{}^2)\,(2mp + (m_a - m)^2)}{p\,(m + m_a)^2}$$

$$c = \frac{2mp + (m_a - m)^2}{p + m - m_a}$$

$$d = \frac{1 - m^2}{2m} \cdot \frac{2mp + (m_a - m)^2}{(p + m - m_a)^2}$$

$$e = s + \frac{m^2 - m^2{}_a}{p + m - m_a}$$

$$u = \frac{p\,(m_a + m)^2}{2mp + (m_a - m)^2}$$

$$v = \frac{2m\,(p + m - m_a)^2}{2mp + (m_a - m)^2}$$

The equivalence of these networks is complete, so that any one may be replaced by the other irrespective of what is connected to their terminals. The process of transforming a network into its equivalent may therefore be carried out on a group of components in the middle of a filter.

In some circuits to be transformed, the component represented by Sy does not appear. This only means that the value of S to be substituted in the equation is zero.

For clarity in the diagrams z is represented by an inductor and y by a capacitor as in the previous papers. The formulae of course hold when z and y are any combination of reactors.

---

* "Double Derived Terminations," *Wireless Engineer*, February and November 1946.

4. Derivation of Filters

Type 1.1

The simplest type of filter with second-order impedances is one having a propagation function equal to half a prototype section. This is derived as shown in Fig. 5.

The filter shown in Fig. 5 (a) consists of half a prototype section joined to half a series m-derived section on each side, the purpose of the latter being to produce the required image impedances. After applying transformation formulae I to the right-hand side of the circuit, the equivalent filter network of Fig. 5 (b) is obtained. A whole derived section may now be detached from the left-hand side of the filter as shown in Fig. 5 (c). The section on the right-hand side will then have a propagation function equal to the prototype half-section and, as may be seen, its image impedances are both of the second order.

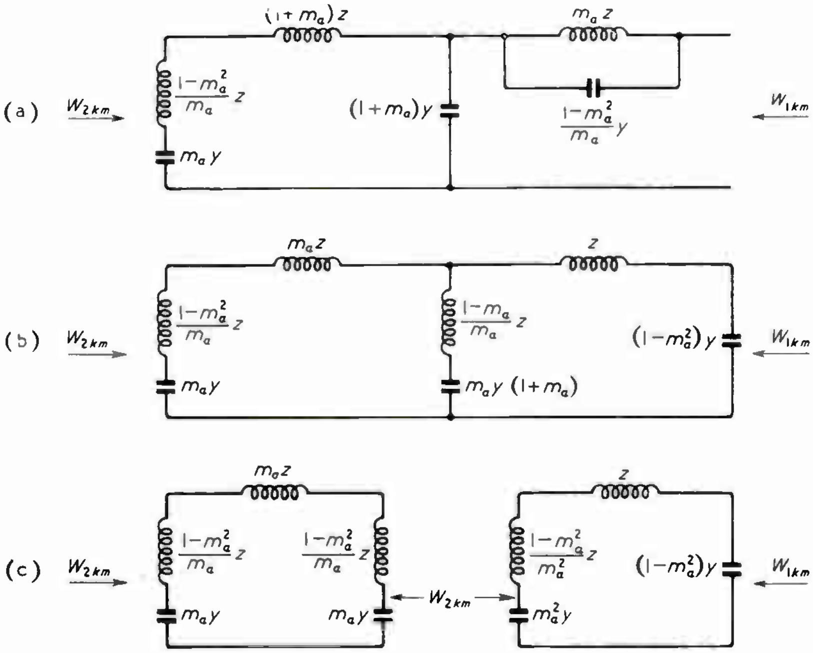


Fig. 5. *Derivation of the simplest 2nd-order impedance section of type* 1.1.

Type 1.2

The next type of filter to be considered is one with a propagation function equal to a full derived section. This filter is developed as shown in Fig. 6. Fig. 6 (a) shows the full section plus the terminating half-sections. Fig. 6 (b) shows the equivalent network obtained by applying transformation formulae II. The removal of a series-derived section from the left-hand side of the network leaves the filter shown in Fig. 6 (c) which is the one required. The values of the components in this filter are given below. A limitation is imposed by the fact that for e to be positive m_1 must be greater than m_a.

In the final network

$$w = \frac{2m_a^2 m_1}{m_a^2 + m_1^2}$$

$$v = \frac{2m_1^3}{m_a^2 + m_1^2}$$

$$f = \frac{(1 - m_a^2)(m_a^2 + m_1^2)}{2m_a^2 m_1}$$

$$c = \frac{m_a^2 + m_1^2}{m_1}$$

$$d = \frac{(1 - m_1^2)(m_a^2 + m_1^2)}{2m_1^3}$$

$$e = \frac{m_1^2 - m_a^2}{m_1}$$

In the second network

$$u = \frac{m_a (m_a + m_1)^2}{m_a^2 + m_1^2}$$

$$b = \frac{(1 - m_a^2)(m_a^2 + m_1^2)}{m_a (m_a + m_1)^2}$$

Type 1.3

A filter of this type designed in the usual manner is shown in Fig. 7 (a). After applying transformation formulae I to the components on the right-hand side, the network shown in Fig. 7 (b) is obtained. This may be transformed into Fig. 7 (c) by applying formulae II to all the components other than those in shunt with the input and output terminals. The removal of a shunt-derived section from the left-hand side gives the required filter as in Fig. 7 (d).

In the final network

$$w = \frac{m_a^2 (m_a^2 + 2m_1 + m_1^2)}{2m_a^2 m_1 + m_a^2 + m_1^2}$$

$$v = \frac{2m_1 (m_a^2 + m_1)^2}{2m_a^2 m_1 + m_a^2 + m^2_1}$$

$$f = \frac{(1 - m_a^2)(2m_a^2 m_1 + m_a^2 + m^2_1)}{m_a^2 (m_a^2 + 2m_1 + m^2_1)}$$

$$c = \frac{2m_a^2 m_1 + m_a^2 + m^2_1}{m_a^2 + m_1}$$

$$d = \frac{(1 - m^2_1)(2m_a^2 m_1 + m_a^2 + m^2_1)}{2m_1 (m_a^2 + m_1)^2}$$

$$e = \frac{m_1 (1 + m_1)}{m_a^2 + m_1}$$

In the third network

$$u = \frac{m_a (1 + m_a)(m_a + m_1)^2}{2m_a^2 m_1 + m_a^2 + m^2_1}$$

$$b = \frac{(1 - m_a)(2m_a^2 m_1 + m_a^2 + m^2_1)}{m_a (m_a + m_1)^2}$$

except that $g = c - m_a$

$$= \frac{m_a^2 - m_a m_1 + m^2_1}{m_1}$$

Denoting the tolerance by t we can proceed to evaluate the limit set upon m_1 in terms of m_a and t.

We have then that

$$w = \frac{2m_a^2 m_1}{m_a^2 + m_1^2} = m_a (1 - t)$$

i.e., $(1 - t)\, m_a^2 - 2m_a m_1 + (1 - t)\, m^2_1 = 0$

$$\text{giving } \frac{m_1}{m_a} = \frac{1 \pm \sqrt{1 - (1 - t)^2}}{(1 - t)}$$

$$= \frac{1 \pm \sqrt{2t - t^2}}{1 - t}$$

Filters, Type 2

In certain cases it may be desirable to have filters with a first-order impedance function at one pair of terminals and a second-order impedance function at the other pair of terminals, the latter impedance being independent of the propagation function. This is not strictly possible, but a solution may be obtained to a limited extent by taking into account the manufacturing tolerances that are allowed in practical filters.

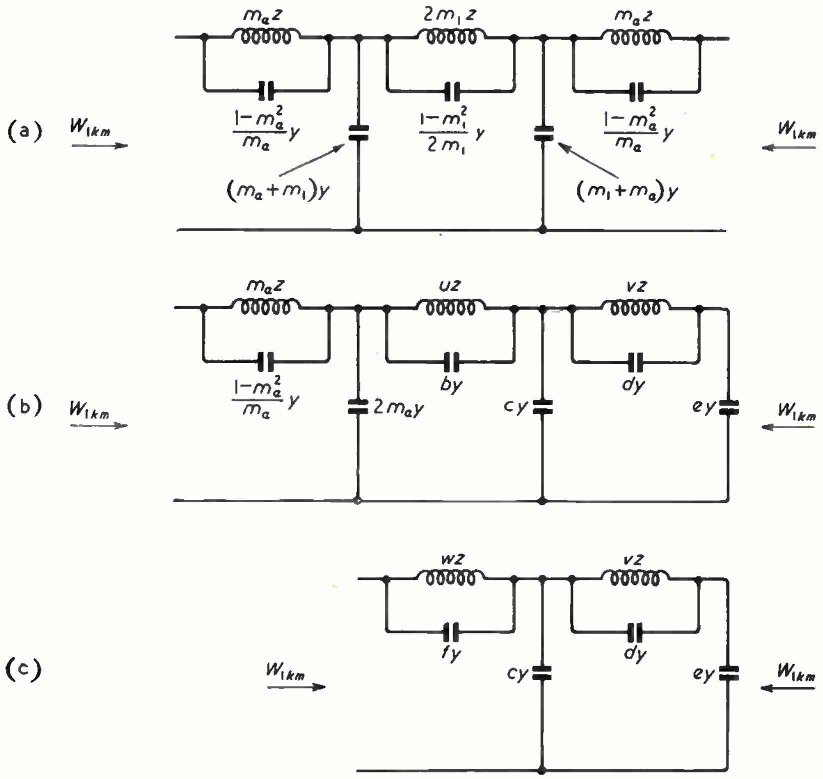


Fig. 6. *Derivation of type* 1.2 *section with 2nd-order impedance.*

Type 2.1

If in filter type 1.2 shown in Fig. 6 m_a is nearly equal to m_1, then w is nearly m_a. If the percentage by which it differs from m_a is tolerable from a practical standpoint a further half-section may be removed from the left-hand side of the filter. This has the effect of making the impedance on this side W_{2k}, which is of the first order. The filter will now be as shown in Fig. 8. The values of the components will be the same as in filter 1.2 Remembering the condition that obtained for filter type 1.2, namely, that $m_1 > m_a$ and approximating to the lowest power of t, we get that

$$m_1 = (1 + \sqrt{2t})\, m_a$$

Taking a practical value of t as up to 1% and m_a as 0·6, then m_1 may take any value between 0·6 and 0·685.

Type 2.2

A similar argument may be applied to the network of Fig. 7 (d). If the value of w is equal to $m_a (1 - t)$, we have this time

$$w = \frac{m_a^2 (m_a^2 + 2m_1 + m^2_1)}{2\, m_a^2 m_1 + m_a^2 + m^2_1} = m_a (1 - t)$$

$$\text{i.e.,}\quad m_a^3 + 2m_a m_1 + m_a m^2_1 = 2m_a^2 m_1 (1 - t) + m_a^2 (1 - t) + m^2_1 (1 - t)$$

$$\text{or}\quad m^2_1 (1 - m_a - t) - 2m_a m_1 (1 - m_a + m_a t) + m_a^2 (1 - m_a - t) = 0$$

$$\text{giving}\ \frac{m_1}{m_a} = \frac{1 - m_a + m_a t \pm \sqrt{(1 - m_a + m_a t)^2 - (1 - m_a - t)^2}}{1 - m_a - t}$$

$$= \frac{1 - m_a + m_a t \pm \sqrt{t (1 + m_a)(1 - m_a)(2 - t)}}{1 - m_a - t}$$

and approximating to the lowest power of t

$$m_1 = \left(1 \pm \sqrt{2t \frac{1 + m_a}{1 - m_a}}\right) . m_a$$

The filter is shown in Fig. 9 and the values of the components are the same as in filter 1.3 except that

$$g = \frac{m_a^2 (1 + m_1)}{m_a^2 + m_1} + m_1 - m_a$$

5. Applications

This type of filter has a rather wider application than that described in the introduction. Some of the circuits involving its use are shown in Fig. 10, where (a) shows a type-2 filter connected to an ordinary ladder filter, (b) shows a type-2 filter fractionally terminated and connected in series or in parallel with its complementary filter, and (c) shows two type-2 filters connected in tandem.

6. Conclusion

The type-1 filters described in this paper pro-

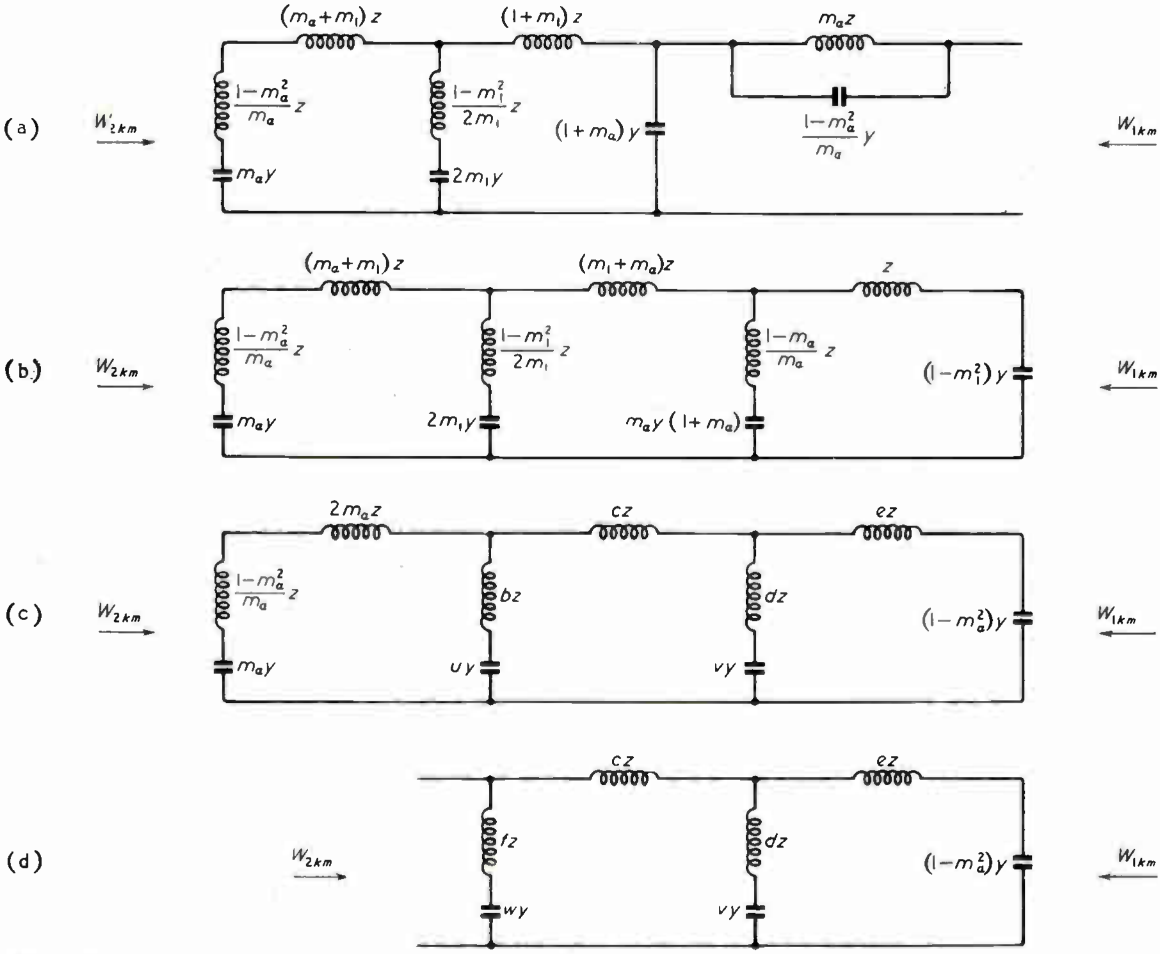


Fig. 7. *Derivation of type 1.3 filter. The usual form is shown at (a). After applying the transformation of formula 1 it takes the form (b), the application of formula 11 then brings it to (c) and the removal of the left-hand section gives the final form (d).*

vide attenuation and impedance functions equal to those of conventional design, but do so with a minimum number of components. The type-2 filters, however, are novel in that their attenuation function is not equivalent to that of conventional filters. In the latter the attenuation is the sum of that of a number of basic half-sections, whereas in the former it includes difference as well as sum. For example, filter type 2.1 was derived from a conventional filter consisting of an m_1 section and an m_a half-section by removing from it a whole m_a section. If the attenuation of the original filter is denoted by $m_1 + \frac{1}{2}m_a$ then that of the final filter will be $m_1 - \frac{1}{2}m_a$. Similarly the attenuation of filter 2.2 is that of an m_1 section plus a half prototype section minus an m_a half-section.

Each of the foregoing filters has, of course, a dual which may be obtained by the well-known method of interchanging inductors and capacitors, and series and parallel connections. Again, although low-pass filters have been depicted throughout, the formulae apply equally well to high-pass or band-pass filters.

Fig. 8 (right). *Type 2.1 filter section.*

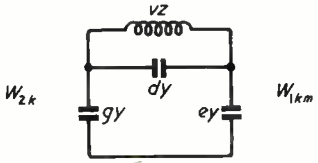
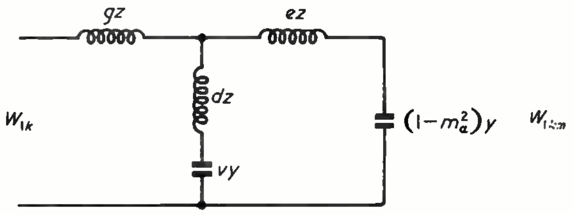


Fig. 9 (below). *Type 2.2 filter section.*



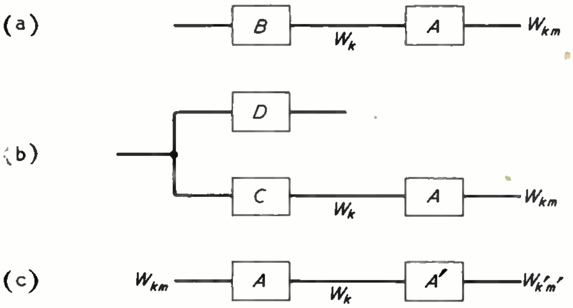


Fig. 10. *Methods of using type-2 filters. A and A′ = type-2 filters; B = ordinary ladder filter; C = fractional termination; D = complementary filter.*

7. Acknowledgment

The author is indebted to Dr. Sturley for his valuable suggestions in the composition of this paper.

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Electrical and Magnetic Units

SIR,—In an Editorial, headed "Electrical and Magnetic Units", in the November issue of *Wireless Engineer*, G. W. O. H. critically questioned the wisdom of the new system of units which O. Löbl proposed in the *Elektrotechnische Zeitschrift* of 1st August, 1951.

Professor Löbl's proposed system of units is unlikely to affect the firmness of the m.k.s. (or Giorgi) system of units which took about 50 years to become established in the minds of the bulk of the electrical-engineering world, and which has already also been accepted by many physicists. However, his arguments to justify the proposal of a new system of units are very interesting because, implicitly, they bring to light an important point in favour of the Giorgi system which seems to have been overlooked in any discussion on systems of units.

The physicists of a hundred years ago found it convenient to use for the magnetic field the notations of magnetic flux density B and magnetic field strength H, and for the electric field the electric flux density D and electric field strength $\mathcal{E}$, on the understanding that in free space $B = H$ and $D = \mathcal{E}$. This convention was a mistake.

The mistake has now been rectified in the Giorgi system by introducing the conceptions of 'magnetic flux density of free space' and 'electric flux density of free space'. Hence, the conceptions of 'magnetic field strength (or magnetizing force)' (H) and 'electric field strength (or electric force)' ($\mathcal{E}$) are put into their proper places and they are given, as they are entitled to possess, their own and proper physical dimensions.

R. FEINBERG.

C.R.T. Laboratory,
Ferranti, Ltd.,
Moston, Manchester.
21st December, 1951.

Impedances in Parallel

SIR,—Referring to the graphical construction for the impedance equivalent to two impedances connected in parallel, may I bring to your notice a graphical solution further to those mentioned in the letters published in your journal, the January, April and July 1951 issues.

This construction is shown in the figure where first the points A and B are found at the intersections of the

World Radio History

DOUBLE-DERIVED TERMINATIONS*

By R. O. Rowlands, B.Sc.

(*The General Electric Co., Ltd.*)

SUMMARY.—A method is described for connecting filters in series or parallel such that the real part of their impedance or admittance function is equivalent to that of a double m-derived section.

Some of the networks obtained are original and have been covered by a patent application.

Introduction

IT is well known that the image impedance of a constant-k type filter varies considerably over the pass range of the filter. When it is desirable that a filter should have a fairly flat impedance characteristic it is generally terminated with one-half of an m-derived section, a value of $m = 0.6$ being usually chosen. With this value of m the impedance only varies about 5 per cent. over 80 per cent. of the pass range.

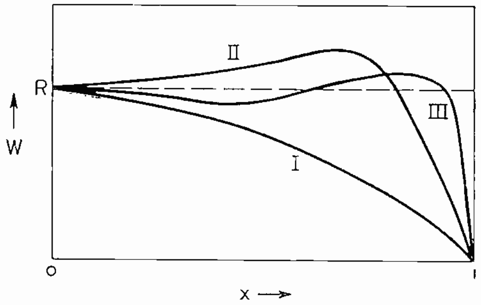


Fig. 1. *Impedance functions with I, Constant-k termination ; II, m-Termination ; and III, mm′-Termination.*

Zobel has shown[1] that by repeated derivation the image impedance can theoretically be made to approach a constant value in an oscillatory manner over as large a part of the pass range as is desired. The higher the degree of derivation, however, the more complicated the filter becomes, so that in practice it is not usual to employ a termination of more than the 2nd degree of derivation. This is called an mm'-termination.

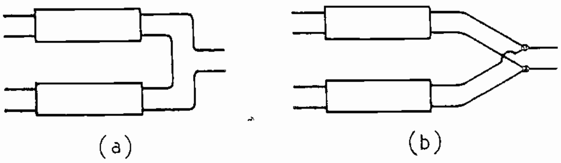


Fig. 2. *Complementary Filters,* (a) *in series, and* (b) *in parallel.*

The behaviour of the impedance function for these three types of terminations is shown in Fig. 1 in terms of x where

$$x = f/f_c \text{ for a low-pass filter.}$$

$$= f_c/f \text{ for a high-pass filter.}$$

$$= \frac{f - \frac{f_m^2}{f}}{f_{c1} - f_{c2}} \text{ for a band-pass filter.}$$

$$= \frac{f_{c1} - f_{c2}}{f - \frac{f_m^2}{f}} \text{ for a band-stop filter.}$$

Then $0 < x < 1$ represents the pass range, and $1 < x < \infty$ represents the stop range.

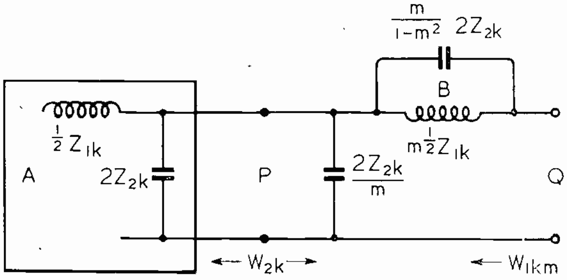


Fig. 3. *A is a composite filter of image impedance* W_{2k}.

Complementary Filters.

These are two filters having substantially the same cut-off frequencies and with the pass range of one corresponding to the stop range of the other. They are connected either in series or parallel at one end, but have their other ends separate as shown in Fig. 2.

The networks (*a*) and (*b*) are inverse networks, i.e. for every property of one network the corresponding property of the other network may be obtained by the application

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* MS. accepted by the Editor, October, 1945.

of the principle of duality.[2] In the following discussion, therefore, only one of the above cases will be considered in proving a particular relationship.

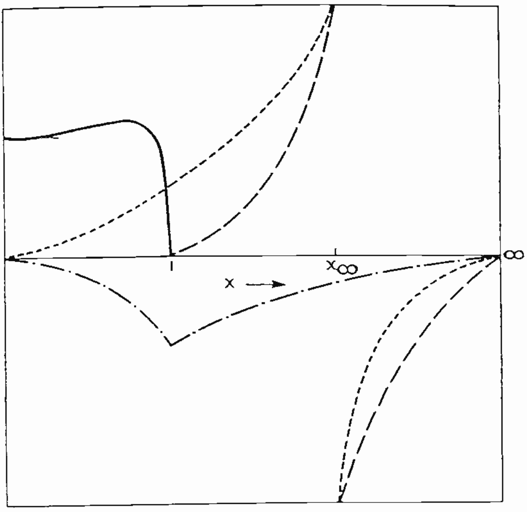
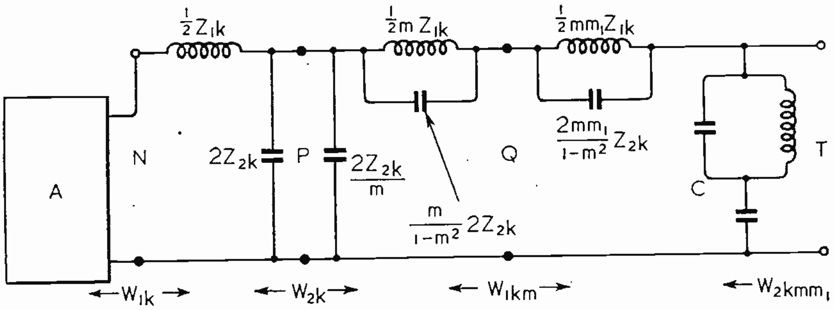


Fig. 4 (Above). *The image impedance of the circuit of Fig. 3 at the terminal Q.*

—— *Real part of* W_{1km}.
– – – *Imaginary part of* W_{1km}.
- - - - - jB.
—·—·— *Imaginary part of* $W_{1km} - jB$.

Fig. 5 (Right). *This circuit is the network of Fig. 3, with a double-derived half-section added at terminals Q.*



Also, although all the formulae will be worked out in terms of Z_{1k} and Z_{2k}, the series and shunt arms of a general filter, the figures will be drawn for the case of a low-pass filter as this will make the argument easier to follow.

Fractional Termination

There are a number of different ways of regarding fractional terminations.[3] For the purpose of this article it is convenient to regard them as being formed in the following manner. In Fig. 3, let A be a composite filter having an image impedance W_{2k} at terminals P. If a shunt-derived half-section is connected in tandem with it the image impedance at the new terminals Q is W_{1km} shown in Fig. 4.

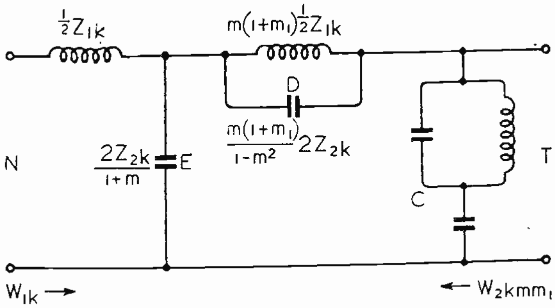


Fig. 6. *The network of Fig. 5 can be reduced to the form shown here.*

If it is to be suitable for connecting in series with its complementary filter at terminals Q, its impedance in its stop range must be low in order not to interfere with the performance of its complementary filter in the latter's pass range.

The impedance function is resistive in the pass range and reactive in the stop range. It has a pole (i.e. its value is infinity) at one frequency, and so in this form it is unsuitable for connecting in series with its complementary filter at these terminals. The pole, however, is due entirely to the last series arm B of the network, and as B is composed of pure reactances the real part of the impedance looking in at Q is unchanged if B is removed.

i.e. if $W_{1km} = R + jX$
where $R\,(x) = 0$ for $1 < x < \infty$
and $X\,(x) = 0$ for $0 < x < 1$
then $W'_{1km} = W_{1km} - jB$
$= R - j(B - X)$

The effect of removing B is shown in Fig. 4. It will be seen that the imaginary part of W'_{1km} is fairly symmetrical about $x = 1$. The complementary filter similarly terminated will have a similar impedance function except that the imaginary part will be of opposite sign, and so when the two are connected in series the imaginary parts of the impedance at the common terminals largely cancel each other out, the extent to which this is achieved depending upon the value of m chosen. The real part of the impedance of each filter will be the same as that of a shunt-derived section.

C

The above process amounts to connecting a reactor in shunt with the output terminals of each filter and since the end shunt arm in each filter is already a similar reactor it merely amounts to modifying the value of this. This is known as a fractional termination. In the case of filters connected in parallel fractional termination consists of increasing the end series arm by $\frac{1}{2}mZ_{1k}$.

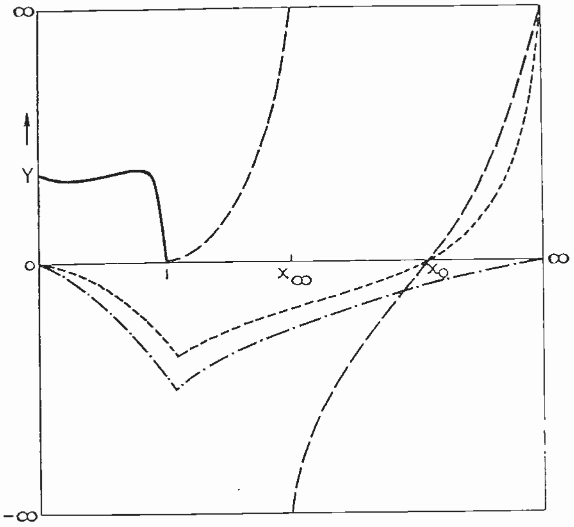


Fig. 7. *The admittance function at terminals T of the network of Fig. 6.*
——— *Real part of admittance at T.*
– – – – *Imaginary part of admittance at T.*
- - - - - - *Imaginary part of admittance at T with C removed.*
–·–·– *Imaginary part of admittance at T with C and F removed.*

Double-Derived Terminations

Having proceeded thus far the next logical step is to attempt to improve the impedance function still further by connecting a double-derived half-section to terminals Q of Fig. 3 and repeating the method which produced the fractional termination. If the end half-section of A is regarded as being outside A then the network will be as shown in Fig. 5. As we are from now on concerned with the network between terminals N and T, terminals P and Q may be ignored and the network becomes that of Fig. 6.

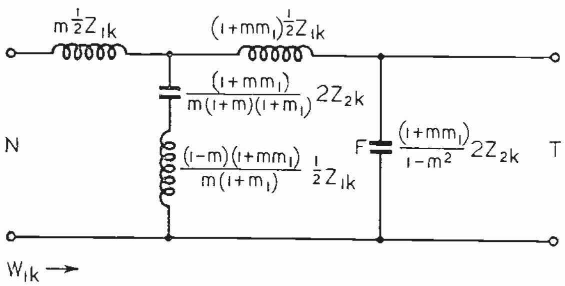

$m\frac{1}{2}Z_{1k}$
$(1+mm_1)\frac{1}{2}Z_{1k}$
$\frac{(1+mm_1)}{m(1+m)(1+m_1)}2Z_{2k}$
$\frac{(1-m)(1+mm_1)}{m(1+m_1)}\frac{1}{2}Z_{1k}$
F
$\frac{(1+mm_1)}{1-m^2}2Z_{2k}$
N
T
W_{1k} →

Fig. 8. *The removal of the arm C from Fig. 6 reduces the circuit to this form.*

It will be seen that the network of Fig. 6 differs from that of Fig. 3 in that it is terminated on the right in a shunt arm instead of a series arm. It is therefore necessary to consider the admittance at terminals T rather than the impedance.

This admittance function (shown in Fig. 7) has two poles, one at $x = x_\infty$ and the other at $x = \infty$. The pole at $x = x_\infty$ is due entirely to the series resonance of the arm C, and the removal of C leaves the real part of the admittance unchanged but does not, however, remove the pole at $x = \infty$. (In other words the impedance at T with C removed is still zero at $x = \infty$, due to components D and E).

It can be shown, however (see Appendix I) that the network of Fig. 6 with the arm C removed is equivalent to that of Fig. 8. Here the reactor F $(= D + E)$, which contributes to the pole in the admittance function at $x = \infty$, appears in shunt across terminals T, and so can be removed without affecting the real part of the function. The resulting network is the required termination; it can be connected to a filter of impedance W_{1k} at terminals N and connected in parallel with its complementary filter similarly constructed, at terminals T. The real part of the admittance at T will be equivalent to that of a filter terminated in an mm'-half section while the imaginary components being of opposite sign in each filter will largely cancel each other out.

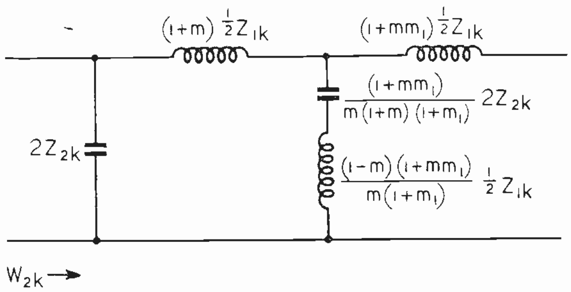

$(1+m)\frac{1}{2}Z_{1k}$
$(1+mm_1)\frac{1}{2}Z_{1k}$
$2Z_{2k}$
$\frac{(1+mm_1)}{m(1+m)(1+m_1)}2Z_{2k}$
$\frac{(1-m)(1+mm_1)}{m(1+m_1)}\frac{1}{2}Z_{1k}$
W_{2k} →

Fig. 9. *The addition of a half-prototype section to Fig. 8 brings the network to this form.*

Preferred Form

It will be seen that the termination of Fig. 8 with the reactor F omitted consists of three inductors and one capacitor in the case of a low-pass filter, and three capacitors and one inductor in the case of a high-pass filter. Since capacitors are in general more economical than inductors, it would be preferable to have an equivalent form of the low-pass termination involving a smaller number of inductors.

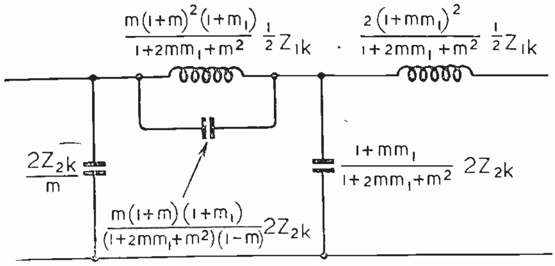

$\frac{m(1+m)^2(1+m_1)}{1+2mm_1+m^2}\frac{1}{2}Z_{1k}$
$\frac{2(1+mm_1)^2}{1+2mm_1+m^2}\frac{1}{2}Z_{1k}$
$\frac{2Z_{2k}}{m}$
$\frac{1+mm_1}{1+2mm_1+m^2}2Z_{2k}$
$\frac{m(1+m)(1+m_1)}{(1+2mm_1+m^2)(1-m)}2Z_{2k}$

Fig. 10. *This diagram is an equivalent to that of Fig. 9, but requires fewer inductors.*

If half a prototype-section is connected to terminals N of the termination, the network of Fig. 9 is obtained. This can be shown (see Appendix II) to be equivalent to the network of Fig. 10, which fulfils the above requirement.

Acknowledgments

The author wishes to thank a number of colleagues for their suggestions regarding the presentation of the above paper, and the management of the G.E.C. for permission to publish it.

REFERENCES

[1] *B.S.T.J.*, April 1931. The notations used by Zobel will be used throughout the present article.

[2] See e.g. Guillemin's "Communication Networks," Vol. II, page 246.

[3] See e.g. Guillemin's "Communication Networks," Vol. II, p. 356, and Starr's "Electric Circuits and Wave Filters," p. 269.

[4] See Guillemin's "Communication Networks," Vol. II, p. 229.

APPENDIX I

If the above networks are equivalent with respect to their input and output terminals then, using the property that the impedances at these terminals are invariant, we have the following transformation equations in matrix form.[4]

For $\frac{1}{2}Z_{1k}$

$$\begin{Vmatrix} r+t & -t & 0 \\ -t & s+t & 0 \\ 0 & 0 & 0 \end{Vmatrix} = \begin{Vmatrix} 1 & x & 0 \\ 0 & y & 0 \\ 0 & z & 1 \end{Vmatrix} \begin{Vmatrix} c & 0 & 0 \\ 0 & d & -d \\ 0 & -d & d \end{Vmatrix} \begin{Vmatrix} 1 & 0 & 0 \\ x & y & z \\ 0 & 0 & 1 \end{Vmatrix} = \begin{Vmatrix} c+dx^2 & dxy & -dx(1-z) \\ dxy & dy^2 & -dy(1-z) \\ -dx(1-z) & -dy(1-z) & d(1-z)^2 \end{Vmatrix} \quad \ldots \quad \ldots \quad (1)$$

And for $2Z_{2k}$

$$\begin{Vmatrix} p & -p & 0 \\ -p & p+q & -q \\ 0 & -q & q \end{Vmatrix} = \begin{Vmatrix} 1 & x & 0 \\ 0 & y & 0 \\ 0 & z & 1 \end{Vmatrix} \begin{Vmatrix} a & 0 & -a \\ 0 & b & 0 \\ -a & 0 & a \end{Vmatrix} \begin{Vmatrix} 1 & 0 & 0 \\ x & y & z \\ 0 & 0 & 1 \end{Vmatrix} = \begin{Vmatrix} a+bx^2 & bxy & bxz-a \\ bxy & by^2 & byz \\ bxz-a & byz & bz^2+a \end{Vmatrix} \quad \ldots \quad \ldots \quad (2)$$

Equating coefficients we have

$$\left.\begin{array}{ll} \text{From (1) Row 3} & z = 1 \\ \text{From (2) Row 1 Col. 3} & x = \dfrac{a}{b} \\ \text{From (2) Row 1} & p - p = 0 \text{ gives} \\ & a + \dfrac{ba^2}{b^2} + \dfrac{bay}{b} = 0 \\ \text{i.e.} & y = -\dfrac{b+a}{b} \end{array}\right\} \quad (3)$$

so

$$\left.\begin{aligned} p &= \frac{a}{b}(a+b) \\ q &= a+b \\ t &= \frac{da\,(a+b)}{b^2} \\ r &= c - \frac{da}{b} \\ s &= \frac{d\,(a+b)}{b} \end{aligned}\right\} \quad \ldots \quad \ldots \quad (4)$$

but

$$\left.\begin{aligned} a &= \frac{1}{1+m} \\ b &= \frac{m\,(1+m_1)}{1-m^2} \\ c &= 1 \\ d &= m\,(1+m_1) \end{aligned}\right\} \quad \ldots \quad \ldots \quad (5)$$

therefore

$$p = \frac{1-m^2}{m(1+m_1)\,(1+m)} \times \frac{1+mm_1}{1-m^2} = \frac{1+mm_1}{m(1+m_1)\,(1+m)}$$

$$q = \frac{1-m+m+mm_1}{1-m^2} = \frac{1+mm_1}{1-m^2}$$

$$t = \frac{m(1+m_1)}{1+m} \times \frac{1+mm_1}{1-m^2} \times \frac{(1-m^2)}{m^2(1+m_1)} = \frac{(1-m)(1+mm_1)}{m\,(1+m_1)}$$

$$r = 1 - \frac{m(1 + m_1)}{1 + m} \times \frac{(1 - m^2)}{m(1 + m_1)} = m$$

$$s = \frac{m(1 + m_1)(1 + mm_1)}{1 - m^2} \times \frac{1 - m^2}{m(1 + m_1)}$$

$$= 1 + mm_1 \qquad \ldots \quad \ldots \quad (6)$$

APPENDIX II

If a reactor $e \,.\, 2z_{2k}$ is connected across the right-hand terminals of each of the equivalent networks of Fig. 11, then equations (4) remain unchanged except that

$$\frac{1}{q'} = \frac{1}{q} + \frac{1}{e} = \frac{1}{a+b} + \frac{1}{e} \qquad \ldots \quad \ldots \quad (4\,a)$$

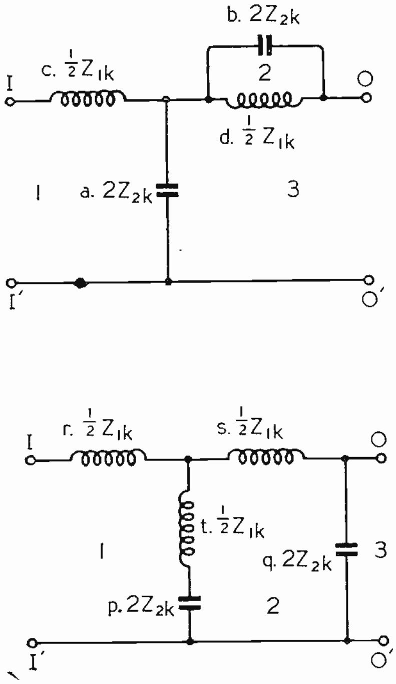


Fig. 11.

and so, solving for a, b, c, d, and e in terms of p, q', r, s and t by means of equations 4 and 4a, we obtain

$$\left.\begin{aligned} a &= \frac{ps}{s+t} \\ b &= \frac{ps^2}{t(s+t)} \\ d &= \frac{-s^2}{s+t} \\ c &= \frac{st}{s+t} + r \\ e &= \frac{pq'\,s}{ps - q'\,t} \end{aligned}\right\} \qquad \ldots \quad \ldots \quad (7)$$

The values of the elements of the network of Fig. 10 can then be obtained from those of Fig. 9 by means of these equations.

CORRESPONDENCE

Pulse Modulation

To the Editor, " Wireless Engineer."

SIR,—The recent proposal by Messrs. Pye, Ltd., of the so-called " videosonic " method of transmitting a television sound channel raises an important issue which appears to be somewhat inexcusably elided in their publications to date ; namely, what audio-frequency band does such a system admit ?

While the answer to this question is almost obvious for the case of a single sinusoidal component of modulation, it is not so obvious whether any additional restrictions arise when a complex wave form is considered ; that is to say, there is no *a priori* guarantee that the system admits of linear superposition. Further, it is not entirely clear whether the audio band restriction has to occur at the transmitter or at the receiver or at both.

In the proposed system, the sound channel is handled by means of a width-modulated pulse transmission, in which the pulses occur during the normal line-sync intervals. Thus the system belongs to a class which may more generally be termed " strobosonic " ; the characteristic feature of this class being that the sound waveform is reconstructed from a succession of extremely short glimpses at the original. The signal can thus be conveniently expressed, in terms of a notation which the writer proposed some years ago.* as follows :—

$$\{ a_0 + a_1 \cos p_1 t + a_2 \cos p_2 t + \ldots \} \operatorname{col} qt,$$

where the term in brackets represents the complex modulating wave form and col qt is the repetitive impulsive function of afrequency q, having the definition

$$\operatorname{col} qt = 0 \text{ for } qt \neq 2n\pi$$
$$\operatorname{col} qt \to \infty \text{ for } qt \to 2n\pi$$

the nature of the infinity being defined by

$$\lim_{\epsilon \to 0} \int_{n\pi - \epsilon}^{2n\pi + \epsilon} \operatorname{col} \theta \,.\, d\theta = 2\pi$$

Using the theorem

$$\cos pt \operatorname{col} qt = \sum_{-\infty}^{\infty}{}_{s} \cos (p - sq)\, t$$

we see that the signal may be written

$$a_0 \sum_{-\infty}^{\infty}{}_{s} \cos sqt + a_1 \sum_{-\infty}^{\infty}{}_{s} \cos (p_1 - sq)t + \ldots$$

The wanted terms in the expression are those corresponding to $s = 0$. All the other terms, which are in the nature of image frequencies with respect to the strobe frequency and its multiples, have to be excluded by filtering.

It is thus clear :

(*a*) That there is an absolute limit for the audio band at half the strobe frequency.

(*b*) That restriction to this band must be carried out by filtering both at the transmitter and at the receiver, and

(*c*) That the linear superposition principle has suggested. With his so-called random alignment of cubes the flux density is the same everywhere, and it is not surprising that it gives results entirely different from those given by the other two hypotheses. It is reassuring to note that the other two hypotheses, although so fundamentally different, give results which differ so little for practical values of the space-factor.

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* *Wireless Engineer*, November 1938.

The large discrepancies between these calculated values and the experimental results obtained by Legg and Given are very surprising but very satisfying from a practical point of view. It was rather a poor outlook for dust cores when it was thought that the permeability, even with the best material, could not exceed about 50. In a Special Report issued by the Radio Research Board in 1934 and published by the Stationery Office, entitled "Magnetic Materials at Radio Frequencies," it was stated that "all the effective permeability formulae . . . . agree in the main conclusion . . . that a figure of about 80 is the highest practically attainable, even with particles of the highest permeability." If the figures given by Legg and Given are reliable a dust-core permeability of over 150 can now be obtained with particles of which the permeability is only 220, but we must confess that we are not convinced that such a result can be obtained if the particles are properly insulated from each other. It could be obtained if the iron were linked up in the form of threads in the direction of the magnetic field. To take two extreme cases, laminations normal to the flux, i.e. in effect as in Lamson's hypothesis, would have a resultant permeability equal to $\mu_1/[\mu_1 - p\,(\mu_1 - 1)]$ which for $\mu_1 = 220$ and $p = 0.9$ gives a permeability of 9.6, whereas for the same quantity of iron, that is, the same value of p, arranged in the form of fine iron wires in the direction of the field, the resultant permeability would be simply $1 + p(\mu_1 - 1)$ which is equal to 198.1. Our formula for cubes gives 24 which seems to us to be more probable than the figure 128.3 given by Legg and Given unless the iron is largely in the form of fine threads in the direction of the field — a very desirable achievement.

G. W. O. H.

DOUBLE-DERIVED TERMINATIONS*

An Extension to Filter Theory

By R. O. Rowlands, B.Sc.

(The General Electric Co., Ltd., Coventry).

Introduction

IN an earlier paper[1] the author described a method of terminating complementary filters at their common ends in such a way that the real part of the image impedance at these terminals is that of a double-derived section.

When the impedance function is prescribed, however, the attenuation function of the terminating section is determined.

In the present paper the method is extended to allow more latitude in the choice of attenuation function when the impedance function is prescribed.

Composite Filters

A composite filter is built up as a ladder network by connecting a number of conventional derived-sections in tandem with the termination. Since the actual impedance looking in at one end of the filter depends upon the degree of matching at the other end, the filter will normally be terminated with a double-derived half-section at the end remote from the common terminals. The "m" values chosen for the intermediate sections are such that the composite filter gives the required attenuation in its stop range. A typical filter is shown in Fig. 1, $W'_{1km_2m_1}$ being the impedance function whose real part is equal to $W_{1km_2m_1}$.

The attenuation provided by the termination on the right-hand side is equivalent to that of a conventional section with m equal to m_2, while that provided by the termination on the left-hand side is that of two half-sections with m values equal to m_2, and m_1 times m_2. The filter, therefore, contains $1\frac{1}{2}$ sections with m equal to m_2. This is uneconomical. The problem is therefore how to get rid of a section with m equal to m_2

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* MS accepted by the Editor, April 1946.

from the filter without affecting the impedance characteristic.

The formulae given in Appendix I enable us to interchange the m_2 and m_3 sections of the filter of Fig. 1. The m_2 section will now be adjacent to a conventional m_4 section, and so they can be interchanged by the application of the same formulæ. By $n - 2$ repeated applications the m_2 section can be moved so that it is adjacent to the conventional m_2 half-section preceding the double-derived half-section.

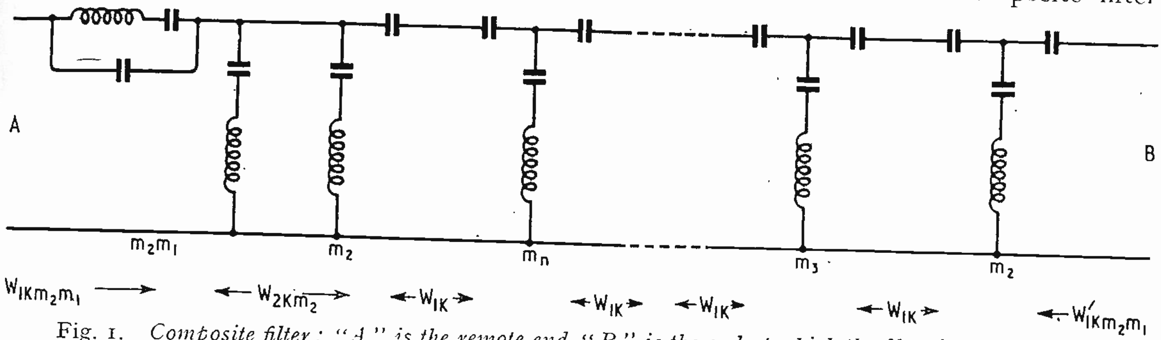

Fig. 1. *Composite filter; "A" is the remote end, "B" is the end at which the filter is connected in parallel with its complement.*

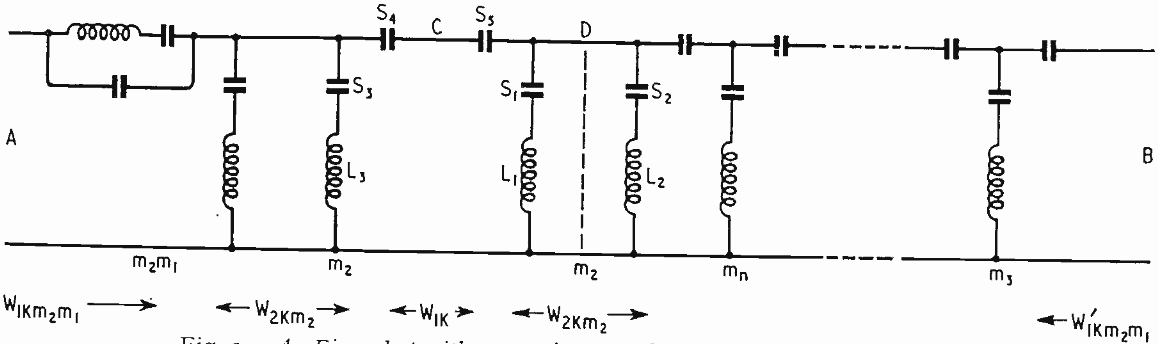

Fig. 2. *As Fig 1, but with m_2 section moved towards the "A" terminals.*

The filter will now be as shown in Fig. 2.

It will be seen that the coefficient of the component S_5 is "*a*," which is always equal to m_2, the value of this coefficient in a conventional section. If the transferred m_2 section, the shunt components of which are L and S, is split along the dotted line D into L_1 and S_1 in parallel with L_2 and S_2 in such a way that L_1 and S_1 are respectively equal to L_3 and S_3, then the image impedance of the half-section comprising S_5, S_1 and L_1 is W_{1k} at the junction C and W_{2km2} at the junction D. But W_{1k} was the image impedance at C before the network was split at D, so by the theorem proved in Appendix II the image impedances looking in both directions at the junction D are equal.

The section comprising L_1, S_1, S_5, S_4, S_3 and L_3 may, therefore, be taken out of the network and the remaining parts joined with perfect impedance matching at the junction. this means that a whole m_2-section has been removed from the network without affecting the latter's impedance characteristic at Terminals A and B. The composite filter may, therefore, be designed to have a prescribed double-derived impedance function but with latitude in the choice of the attenuation function of all but two half-sections which will have m values of m_2 and m_1 times m_2.

A filter made up of shunt-derived sections may be treated in the same way. The coefficients of the components "a," "b," etc., will in this case be the inverse of those for series-derived sections.

Practical Simplification

It may appear at first sight that in a filter with many sections the process of transferring the m_2 section from one end of the filter to the other is long and laborious. In practice this is not the case, for suppose q differs from its value in a conventional section by the factor δ;

$$\text{i.e., } q = \frac{1}{2m_2} + \delta$$

Then
$$u = \frac{2m_3 + (m_3 - m_2)^2\,(1/2m_2 + \delta)}{(m_3 + m_2)^2}$$
$$= \frac{4m_2m_3 + (m_3 - m_2)^2}{2m_2(m_3 + m_2)^2} + \frac{(m_3 - m_2)^2\delta}{(m_3 + m_2)^2}$$
$$= \frac{1}{2m_2} + \left(\frac{m_3 - m_2}{m_3 + m_2}\right)^2\delta$$

The factor δ has been considerably reduced by the transformation. This value u now becomes q for the second transformation and so δ will be reduced a further amount. The best order in which to connect the sections of the composite filter of Fig. 1 so as to reduce δ, in the minimum number of transformations, to such a small value that it may be neglected is, to make

$$|m_3 - m_2| < |m_4 - m_2| < \text{etc.}$$

As soon as δ has reached this value the m_2 section may be removed from the filter without the necessity of any further transformation.

APPENDIX I

If the networks of Fig. 3 are equivalent with respect to their input and output terminals then we have the following transformation matrices.

For $\frac{1}{2}Z_{1k}$,

$$\begin{Vmatrix} a+b, & -b\ , & 0 \\ -b, & b+c+d, & -d \\ 0\ , & -d & , d+e \end{Vmatrix} = \begin{Vmatrix} 1 & x & 0 \\ 0 & y & 0 \\ 0 & z & 1 \end{Vmatrix} \cdot \begin{Vmatrix} f+g, & -g & , \ 0 \\ -g & , g+h+j, & -j \\ 0 & , -j & , j+k \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 & 0 \\ x & y & z \\ 0 & 0 & 1 \end{Vmatrix}$$

$$= \begin{Vmatrix} f+(1-x)g\ , & -g+x(g+h+j), & -xj \\ -yg & , \ y(g+h+j) & , -yj \\ -zg & , z(g+h+j)-j & ,(-z+1)j+k \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 & 0 \\ x & y & z \\ 0 & 0 & 1 \end{Vmatrix}$$

$$= \begin{Vmatrix} f+(1-x^2)g+x^2(h+j)\ , & -yg+xy(g+h+j), & -zg+zx(g+h+j)-xj \\ -yg+xy(g+h+j) & ,y^2(g+h+j)\ , & y(g+h+j)--yj \\ -zg+xz(g+h+j)-xj, & yz(g+h+j)-yj\ , & z^2(g+h)+(1-z)^2j+k \end{Vmatrix} \quad .. \quad .. \quad (1)$$

And for $2Z_{2k}$ substituting 0 for f, h, and k; p for g, and q for j,

$$\begin{Vmatrix} u & -u & 0 \\ -u & u+v & -v \\ 0 & -v & -v \end{Vmatrix} = \begin{Vmatrix} (1-x)^2p+x^2q & -yp+yx(p+q) & -zp+xz(p+q)-xq \\ -yp+xy(p+q) & y^2(p+q) & y(p+q)-yq \\ -zp+xz(p+q)-xq & yz(p+q)-yq & z^2p+(1-z)^2q \end{Vmatrix} \quad .. \quad .. \quad (2)$$

Equation (2) Col. 2 gives

$$-u + (u + v) - v = -yp + yx\,(p + q) + y^2\,(p + q) + yz(p + q) - yq = 0$$

or $(p + q)(x + y + z) = (p + q)$

i.e., $x + y + z = 1$ .. .. (3)

Equation (1) Col. 1 row 3 gives

$$-zg + xz(g + h + j) - xj = 0$$

or $xz(g + h + j) = zg + xj$ .. .. (4)

Equation (2) Col. 1 row 3 gives

$$-zp + xz(p + q) - xq = 0$$

or $xz(p + q) = zp + xq$ .. .. (5)

From (3), (4) and (5)

$$x = \frac{jp - gq}{jp - hq - gq}\ ; \qquad z = \frac{jp - gq}{jp + hp - gq}$$
$$y = 1 - (x + z)$$

If the network on the right-hand side of Fig. 3 is a conventional section joined to a double-derived terminating section then its components values will be as follows:—

$$f = m_3$$
$$g = \frac{1 - m_3^2}{2\,m_3}$$
$$h = m_2 + m_3$$
$$p = 1/(2m_3)$$
$$q = \frac{1 + m_1m_2}{m_2(1 + m_1)(1 + m_2)}$$
$$j = \frac{(1 - m_2)(1 + m_1m_2)}{m_2(1 + m_1)} = (1 - m_2^2)q$$
$$k = (1 + m_1m_2)$$

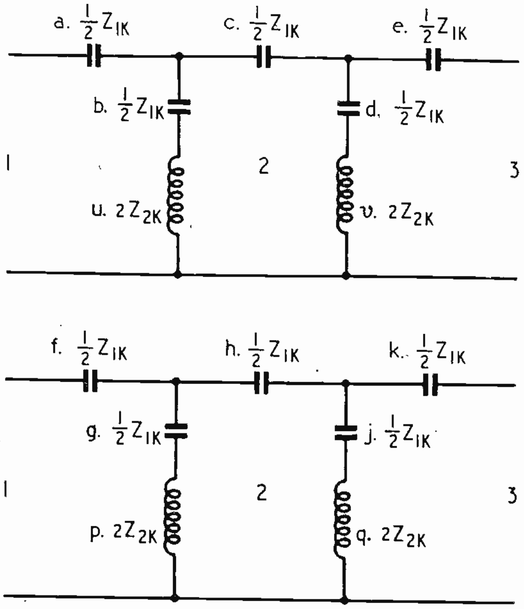

Fig. 3. *Equivalent circuits.*

And so

$$x = \frac{m_2 - m_3}{m_2 + m_3}$$
$$z = \frac{(m_3 - m_2)q}{(m_3 - m_2)q + 1}$$
$$y = \frac{(m_3 - m_2)^2q + 2m_3}{(m_3 + m_2)\{(m_3 - m_2)q + 1\}}$$

The component values of the network on the left-hand side of Fig. 3 are then given by

$$u = \frac{2m_3 + (m_3 - m_2)^2q}{(m_3 + m_2)^2}$$
$$v = \frac{2m_3 + (m_3 - m_2)^2q}{2m_3\{1 + (m_3 - m_2)q\}^2} \cdot q$$
$$a = m_2$$
$$b = \frac{(1 - m_2^2)\{2m_3 + (m_3 - m_2)^2q\}}{(m_3 + m_2)^2}$$
$$c = \frac{2m_3 + (m_3 - m_2)^2q}{1 + (m_3 - m_2)q}$$
$$d = \frac{(1 - m_3^2)\{2m_3 + (m_3 - m_2)^2q\}q}{2m_3\,\{1 + (m_2 - m_2)q\}^2}$$
$$e = k + \frac{(m_3^2 - m_2^2)q}{1 + (m_3 - m_2)q}$$

The above formulae are given in terms of m_2, m_3, q and k, and so they may be applied directly to obtain the first transformation. For the second and subsequent transformations, all that need be done is to substitute m_4 etc. for m_3 and use the appropriate values for q and k.

APPENDIX II

Theorem

If the four-terminal network N_{13} of Fig. 4 can be split into two cascade-connected four-terminal networks N_{12} and N_{23} in such a way that the image impedance at terminals 1, 1′ of both N_{12} and N_{13} is W_1, then the image impedances of both N_{12} and N_{23} at terminals 2, 2′ are equal.

Proof

Let the general circuit parameters of N_{12} and N_{23} in matrix form be $\begin{Vmatrix} A & B \\ C & D \end{Vmatrix}$ and $\begin{Vmatrix} a & b \\ c & d \end{Vmatrix}$ then the parameters of N_{13} are[2],

$$\begin{Vmatrix} A & B \\ C & D \end{Vmatrix} \times \begin{Vmatrix} a & b \\ c & d \end{Vmatrix} = \begin{Vmatrix} Aa + Bc & Ab + Bd \\ Ca + Dc & Cb + Dd \end{Vmatrix}$$

For the network N_{12}, $W_1{}^2 = \dfrac{AB}{CD}$

For the network N_{13}, $W_1{}^2 = \dfrac{(Aa + Bc)\ (Ab + Bd)}{(Ca + Dc)\ (Cb + Dd)}$

So $\dfrac{A\ B}{C\ D} = \dfrac{(Aa + Bc)(Ab + Bd)}{(Ca + Dc)(Cb + Dd)}$.

i.e., $AB\{C^2ab + CD(ad + bc) + D^2cd\}$
$= CD\{A^2ab + AB(ad + bc) + B^2cd\}$

or $ACab(BC - AD) = BDcd(BC - AD)$

i.e., $\dfrac{ab}{cd} = \dfrac{BD}{AC}$

$W'_2 = W_2$

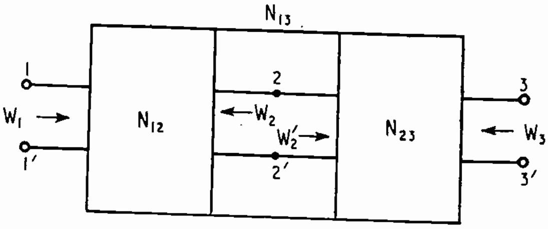

Fig. 4.

Acknowledgments

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REFERENCES

[1] *Wireless Engineer*, Feb. 1946, the same notation will be used except that m and m_1 become m_2 and m_1 respectively in the present article.

[2] See Guillemin's "Communication Networks," Vol. II p. 138, p. 145 and p. 171.

CHARACTERISTICS OF R.F. CABLES*

Determination of Impedance and Propagation Constants

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Introduction

DURING the early stages of the war, and before more refined techniques, such as those described elsewhere by Dr. L. Essen, had been developed, the need arose for measurements at about 600 Mc/s of the phase and attenuation constants and the characteristic impedance of the Polythene insulated cables which were finding wide application in radiolocation equipments. For this purpose a technique was developed which was fundamental in principle and required only simple apparatus, and although it has now been superseded it would still seem to be advantageous for use in the instruction of students. The technique is equally applicable to coaxial- or twin-type cables and the frequency of 600 Mc/s happens to be a very convenient one at which to carry out the measurements.

1. Measurement of the Phase Constant

The length of cable on which measurements are to be made is magnetically coupled to the oscillator at one end and left open at the other, as shown in Fig. 1. A thermojunction, the heater circuit of which is included in the coupling loop, enables the input current to the line to be measured. The procedure consists of cutting short lengths from the open end of the line and observing the input current for each value of line length. A curve of input current against line length is

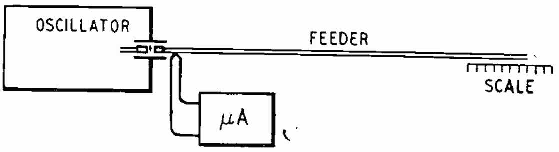

Fig. 1. *Arrangement of apparatus for the measurement of phase constant.*

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